CACTUS

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Some Easter mathematics

In the Western Gregorian Calendar, the date of Easter Sunday is defined as the Sunday following the ecclesiastical Full Moon that falls on or next after March 21. While the pattern of dates so defined usually repeats each 19 years, there is a 0.08 day difference between the cycles. More accurately, the system has a period of 70 499 183 lunations which is about 5 700 000 years: more details are at astro.nmsu.edu/~lhuber/leaphist.html.

This website also provides one version of an algorithm, attributed to Oudin, which is valid for any Gregorian year "y" to calculate the month "m" and day "dy" of Easter Day. I have rewritten the formulae using Java Script (next column). The site goes on to specify that "the variables are all integers and the remainders of all divisions are dropped". The code has therefore been simplified by defining the function D(a,b) which replaces all divisions a/b. Constructs of the form y – 19*(y/19) have been rewritten in a modulus form as y%19.

To get this code working on your machine you have only to copy it into a simple text editor such as Notepad or TextEdit and save it as Easter.html. If you use Word to type the file, save it as a text file and then rename the file as Easter.html. Then open your favourite web browser and load the file Easter.html. If nothing happens adjust your browser preferences so that JavaScript is turned on. When the file loads you should see something like this on your screen.

If you type a year into the first text box and then click the "Do one year" button, the code will provide the month and day for Easter Sunday for that year. If you click "The next year" button the number in the

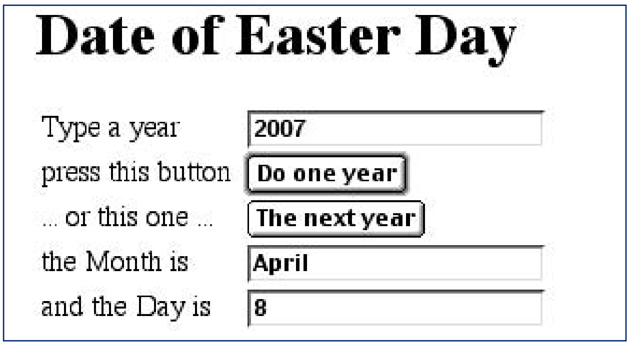
```
<HTML><HEAD>
<TITLE>Date of Easter Day</TITLE>
<SCRIPT LANGUAGE = JavaScript>
<!-- Hide this script from some browsers

function D(a,b)
  {return Math.floor(a/b)}

function inc()
  {var y = 3;
   y = parseFloat(document.es.year.value);
   y = y + 1;
   document.es.year.value = y;
   proc(); }

function proc()
 {var y,c,n,k,i,j,k,l,m,dy = 2;
  var                  mth = "April";
   y = parseFloat(document.es.year.value);
   c = D(y,100);
   n = y%19;
   k = D(c - 17,25);
   i = c - D(c,4) - D(c - k,3) + 19*n + 15;
   i = i%30;
   i = i-D(i,28)*(1-(D(i,28))*D(29,i+1)*D(21-n,11));
   j = y + D(y,4) + i + 2 - c + D(c,4);
   j = j%7;
   l = i - j;
   m = 3 + D(l + 40,44);
      if (m==3) mth = "March";
  dy = l + 28 - 31*D(m,4);
  document.es.month.value = mth;
  document.es.day.value = dy;}

// end of hidden section -->
</SCRIPT></HEAD>
<BODY>
<H1>Date of Easter Day</H1>
<FORM NAME="es">
<TABLE>
 <TR><TD> Type a year </TD>
 <TD><INPUT TYPE="textbox" NAME="year"></TD></TR>
 <TR><TD> press this button </TD>
 <TD><INPUT TYPE="button" VALUE="Do one year"
                 ONCLICK="proc()"></TD></TR>
 <TR><TD> ... or this one ...</TD><TD>
 <INPUT TYPE="button" VALUE="The next year"
                 ONCLICK="inc()"></TD></TR>
 <TR><TD> the Month is </TD>
 <TD><INPUT TYPE="textbox" NAME="month"></TD>
 <TR><TD> and the Day is </TD>
 <TD><INPUT TYPE="textbox" NAME="day"></TD></TR>
</TABLE></FORM>
</BODY></HTML>
```



first text box is incremented and the calculation is repeated. This saves much typing if you wish to work through a nineteen year cycle. This code always gave me the same sample answers that I obtained from more complex code or tables at other internet sites.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Year										Month	Day			
2	y	c	n	k	l	l	l	j		l	m	d			
3															
4	1980	19	4	0	100	10	10	2472	1	9	4	6	-6	Mar 22	23
5	1981	19	5	0	119	29	28	2491	6	22	4	19	-5	Mar 23	42
6	1982	19	6	0	138	18	18	2482	4	14	4	11	-4	Mar 24	76
7	1983	19	7	0	157	7	7	2472	1	6	4	3	-3	Mar 25	102
8	1984	19	8	0	176	26	26	2493	1	25	4	22	-2	Mar 26	112
9	1985	19	9	0	195	15	15	2483	5	10	4	7	-1	Mar 27	134

Adapting the Oudin algorithm to a spreadsheet takes more care with brackets but makes it easier to manage a larger sample of dates. I had found a table of dates starting at 1980 so I built the spreadsheet shown above to calculate Easter Sunday dates from 1980 to 6979. To do this I typed 1980 at cell A4 and 1981 at cell A5. Selecting both cells I used the mouse pointer on the bottom-right tag to copy years down to 6979 at cell A5003. I then entered the following formulae in row 4 which allowed ample room for headings.

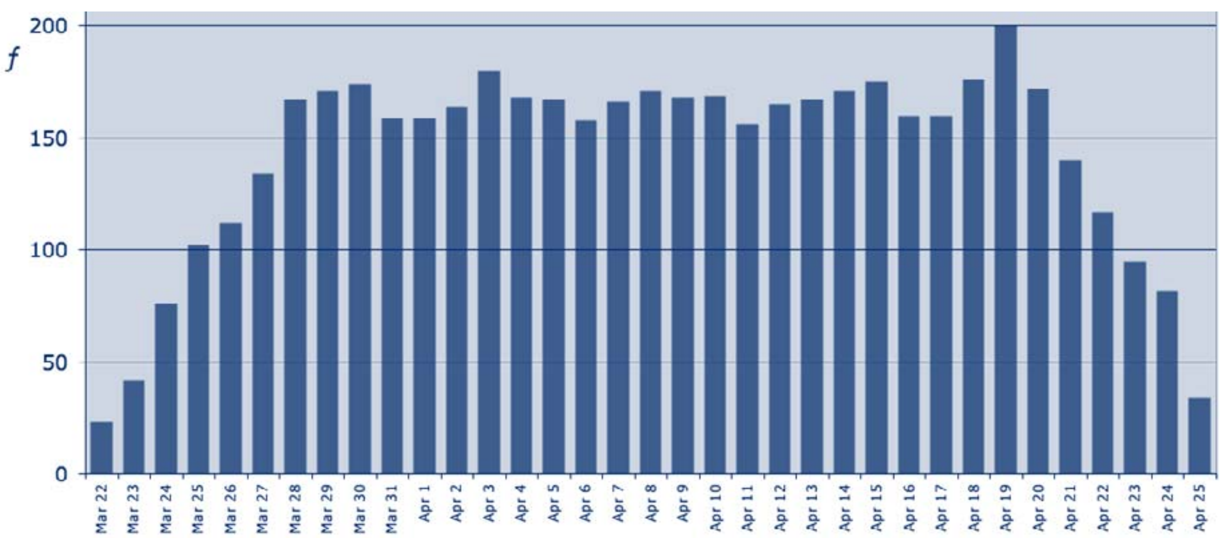
```
B4: =INT(A4/100)
C4: =MOD(A4,19)
D4: =INT((B4-17)/25)
E4: =B4-INT(B4/4)-INT((B4-D4)/3)+19*C4+15
F4: =MOD(E4,30)
G4: =F4-((INT(F4/28)*(1-(INT(F4/28)*  «to next line»
        INT(29/(F4+1))*INT((21-C4)/11)))))
H4: =A4+INT(A4/4)+G4+2-B4+INT(B4/4)
I4: =MOD(H4,7)
J4: =G4-I4
K4: =3+INT((J4+40)/44)
L4  =J4+28-31*INT(K4/4)
```

These formulae are then copied down to row 5003 giving a sample of 5000 years starting at 1980. I was interested to discover whether any particular dates were more likely than others and if some dates were impossible.

When I sorted by column J, I found that the dates sorted one-to-one with column J. Therefore to graph the frequency of the different dates it was easier to work with the integers in column J. However, using Excel to draw the frequency chart shown below is a non-trivial exercise if you want Excel to put dates on the graph correctly.

The first step is to count the different integers in column J. They range from –6 to 28. To do this type –6 at M4, –5 at M5 and then copy down to 28 at cell M38. Then select the cells from O4 to O38 and define a Frequency array formula on that array. As part of the definition we need to enter the data_array as J4:J5003 and then the bins_array as M4:M38. The frequency count of the integers from column J appears in column O.

We now need to re-identify the numbers in the column M bins with a date scale. However, if Excel recognises our scale as dates it will re-sort them incorrectly. To overcome this, select cells N4 to N38 and format them as type text. Then we can type Mar 22 at N4 and Mar 23 at N5 and copy down until cell N13. At N14 we need to start again with April because Excel now recognises the data as text instead of dates. Now use columns N and O with the graphing wizard to draw a graph.



I was surprised how evenly the dates are represented. Obviously the extreme dates are less likely and we get a near trapezoidal distribution. Such a nice outcome depends on considering several thousand years of the full cycle.

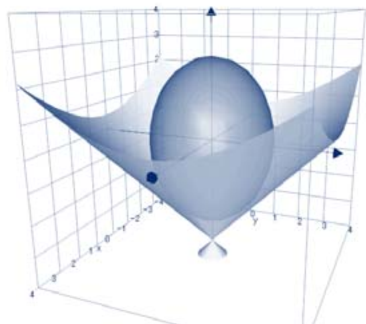
You may have second thoughts about using this material in a class of mixed ethnicity. However, the website at the start of this article has much to say about other calendars in a range of cultures. Here is an opportunity to promote cultural inclusivity in your mathematics classroom. For example, Islamic countries sometimes have different dates for Eid ul Fitr, the end of Ramadam, because this is determined by lunar observation within each country rather than using standardised ecclesiastical tables. Students from Orthodox backgrounds may wish to discover how the date of Easter is calculated in their culture. Some students may wonder why Easter does not occur at the same time as Passover? With about forty calendars used in the World today, a study of cultural and religious dates often leads to interesting mathematical challenges.

Some years ago I reviewed version 2 of the Windows graphing package Autograph. Version 3 has since been released and one of its many extensions is an ability to represent 3D graphs. Recall that Autograph allowed implicit definitions of relationships as well as functions. There is also a feature which allows the user to animate a family sequence of graphs by incrementing the value of a parameter. These same powerful features are now available to construct representations of 3D relationships. I was particularly impressed with the amount of control the user is given over ambient light, background, perspective, colour and the degree of transparency of each surface.

The Autograph toolbar consists of the usual menu bar below which are two rows of icons which give fast access to the most commonly used tools. The first two icons define the page as single variable statistics or two variable statistics and coordinate geometry while the third icon switches on 3D graphing. In the screen dump shown above I have opened the 3D graphing page and then pressed an icon which opens the equation editor.

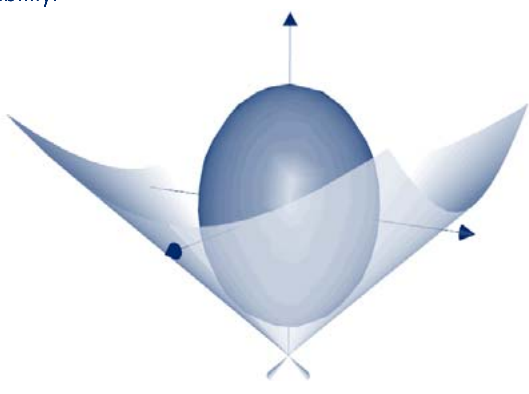
The third row of icons is specific to the type of graph chosen. The first icon restores the x-y-z orientation to the strange Direct-X default which is the same as that shown above. This is not an orientation that I have seen used in Australian classrooms. To work around this, load a blank page, use your mouse to spin the axes around until the orientation looks right and then save the corrected blank file from which students launch. The second icon restores the default ranges and is useful when students have zoomed a 3D structure until it is unrecognisable. The 3rd, 4th and 5th icons select a black, neutral or white background and the next two toggle the grid and the axes. How much you can adjust lighting will depend on the Direct-X capabilities of your graphics card.

The equation editor is particularly user friendly, striking a practical balance between ease of use and the appearance of expressions. For example, some other editors use x^2 to signify x^2. The Autograph editor has buttons which allow you to easily insert special characters such as powers, inequalities and Greek symbols.



The graph shown above is taken from an Autograph web site and is designed to represent an Easter egg on a conical stand. This is formed as a combined graph of the implicit cartesian relationship $x^2/4 + y^2/4 + 5z^2/36 = 1$ represented on the same 3D axes as the polar function $r = z + 3.5$.

In the graph shown below, the grid has been switched off and only the stand has been given some transparency. I know of no other graphing package with this power and flexibility.



From Helen Prochazka's

Scrapbook

The importance of proofs

A theorem is a result which has been proved using mathematical arguments. Proofs are the basic modules that make up mathematics. They give mathematics its great strength. Once something has been mathematically proved it becomes a mathematical truth that cannot be undone. R. Graham has estimated that upwards of 250 000 mathematical theorems are published each year

The ancient Greek Thales (c.624–548BC) is regarded as the first mathematician because he was the first person to insist that a mathematical result had to be proved before it was accepted. His pupil Pythagoras continued this approach. Later Euclid's definitive work, The Elements, became the enduring blueprint of how mathematics should be constructed.

There is something about proving Pythagoras' Theorem that fascinates maths enthusiasts. Thousands of proofs have been devised. Albert Einstein discovered one when he was a teenager. Leonardo da Vinci had a proof. A retired Queensland mathematics teacher, Garnet Greenbury, has written more than 70 proofs.

That queer quantity 'infinity' is the very mischief, and no rational physicist should have anything to do with it. Perhaps that is why mathematicians represent it with a sign like a love-knot.

Sir Arthur Eddington, 20 century astronomer

Investigation Ideas

- Choose three numbers. Make all the different fractions including those with the same numerator and denominator. What is always true about the order of the fractions. Investigate using 4 numbers and maybe five numbers.

- In how many different ways can you cut a 4 × 4 geoboard into congruent quarters if the cuts are straight lines from one nail to another? In how many ways can you cut a square into congruent quarters with straight cuts?